

Truncated apodizers for engineering the point spread function of optical systems

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Abstract—Efficient truncated apodizers have been presented for modulating the point spread function (PSF) of the optical system in the presence of a high level of primary aberrations. It has shown that the resolution of the apodized optical system is increased with the selected degree of apodization β (a shaded region across the aperture function) and the central obstruction ratio ϵ . Furthermore, it is evidenced by the presence of a narrow central lobe and minimal sidelobes in the resulting PSF intensity profile curves and the presence of a steep dip in the resulting composite intensity distributions of two overlapping point sources that are mutually incoherent.

Keywords—Apodization, point spread function (PSF), two-point resolution, aberrations, defocusing

I. INTRODUCTION

The apertures with different transmission functions play a vital role in modifying the point spread function (PSF) of optical systems realized in science and engineering applications. The PSFs with improved intensity distributions are the most desirable in numerous research applications [1-10]. Recently, several studies have paid attention to developing this property [1-3]. An aberrated optical system results in the PSF with non-zero first minima, enhanced sidelobes due to the displacement of internal energy of the central lobe, widened central lobe, and shifted first minima positions, which represent the fundamental problem in aberrated optical systems [1-3]. The aberrated optical system and efficient apodization techniques eliminate the well-known effects induced by various optical aberrations [1-10]. However, it varies from one optical system to another and strongly depends on the coherence conditions and apodization mechanisms to be applied [1,2,6]. The proposed truncated apodizers can modify the focal properties of an aberrated optical system, such as suppressing sidelobe levels, narrowing the central lobe, leading to low FWHM for the PSF of aberrated optical systems. The proposed truncated apodizers are helpful to improve the axial, lateral resolution of confocal scanning and spectroscopic observations. The current study is instrumental in interpreting the interference of intensity distributions of two-point sources under incoherent light illumination, i.e., resolving a composite image of two overlapping point sources with equal intensities and achieve superresolution property. For example, truncated apodizers effectively resolve nearest biological structures or distant objects (interstellar objects, stars) formed by optical systems.

II. THEORY

A. Formulation: Point Spread Function (PSF)

Following scalar wave diffraction theory of aberrated

optical systems, a monochromatic plane wave incident on the truncated apodizer yield diffraction, and that converges at the focal point. An iterative numerical integration method developed and applied to calculate the resultant intensity distribution of the PSF, which is proportional to the Fourier transform of the light field distribution across the apodizing pupil plane. The general expression for the amplitude distribution of light field in the focal region of the optical system obtained, thus [1-3]:

$$B(Z) = 2 \int_{\epsilon}^1 f(r) \exp(-i(\phi_a)) J_0(Zr) r dr \quad (1)$$

$$\phi_a = \exp \left[-i(\phi_d \frac{r^2}{2} + \phi_s \frac{r^4}{4}) \right] \quad (2)$$

In equation (1), $f(r)$ is the Gaussian-type apodization function with the apodization controlling parameter β . Note that ϵ is the central obstruction ratio, Z is the reduced dimensionless diffraction coordinate. Here J_0 represents the Bessel function of the first kind and zero-order. The parameter ‘ r ’ is the radial coordinate in the pupil plane, and ϕ_a is the aberrations function. However, primary monochromatic aberrations considered in this study are the defocusing effect (ϕ_d) and the primary spherical aberration (ϕ_s), as presented in equation (2). The resultant intensity PSF of the aberrated optical system is measured from equation (3).

$$I(Z) = |B(Z)|^2 \quad (3)$$

B. Formulation: Two-point resolution

$$B(Z \pm W) = 2 \int_{\epsilon}^1 f(r) \exp(-i(\phi_a)) J_0[(Z \pm W)r] r dr \quad (4)$$

$$I(Z) = |B(Z - W)|^2 + \delta |B(Z + W)|^2 + 2\sqrt{\delta} \mu(Z_0) |B(Z - W)| |B(Z + W)| \quad (5)$$

Equation (5) is the general expression for the composite image irradiance distribution from two overlapping point sources formed by an optical system in the presence of high defocusing ($\phi_d = 2\pi$) and spherical aberration ($\phi_s = 2\pi$) [1, 2].

In equation (5), δ is the intensity ratio of two overlapping point sources, where $\delta = 1$ represents two-point sources with equal intensities. Over here, $\mu(Z_0)$ denotes the main part of the complex degree of coherence illumination, and Z is the dimensionless diffraction variable. Thus, $B(Z+W)$ and $B(Z-W)$ are amplitude responses of the aberrated optical system corresponding to the two-point sources mutually incoherent, located at the distance of $Z_0/2$ on both sides of the optical axis.

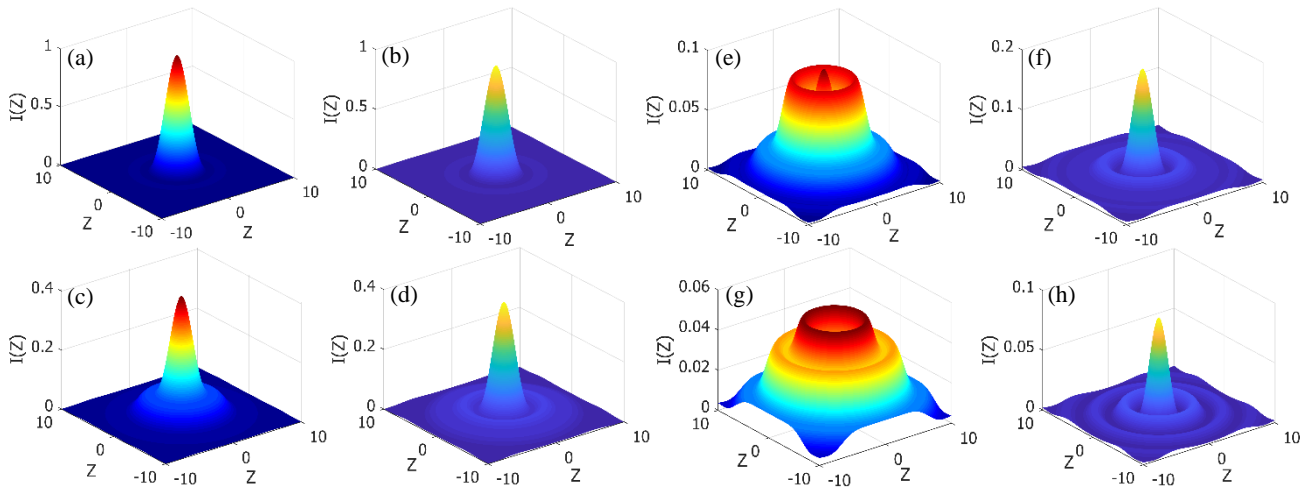


Fig 1. PSF intensity distributions formed by pure optical systems (a) Airy PSF (b) PSF formed by the aperture with $\varepsilon = 0.2$ (20% area of the aperture is truncated by the mask). PSF intensity distributions formed by defocused optical systems (c) $\phi_d = 2\pi$ (d) $\phi_d = 2\pi$, $\varepsilon = 0.4$ (40%) (e) $\phi_d = 3\pi$ (f) $\phi_d = 3\pi$, $\varepsilon = 0.6$ (60%) (g) $\phi_d = 4\pi$ (h) $\phi_d = 4\pi$, $\varepsilon = 0.8$ (80%).

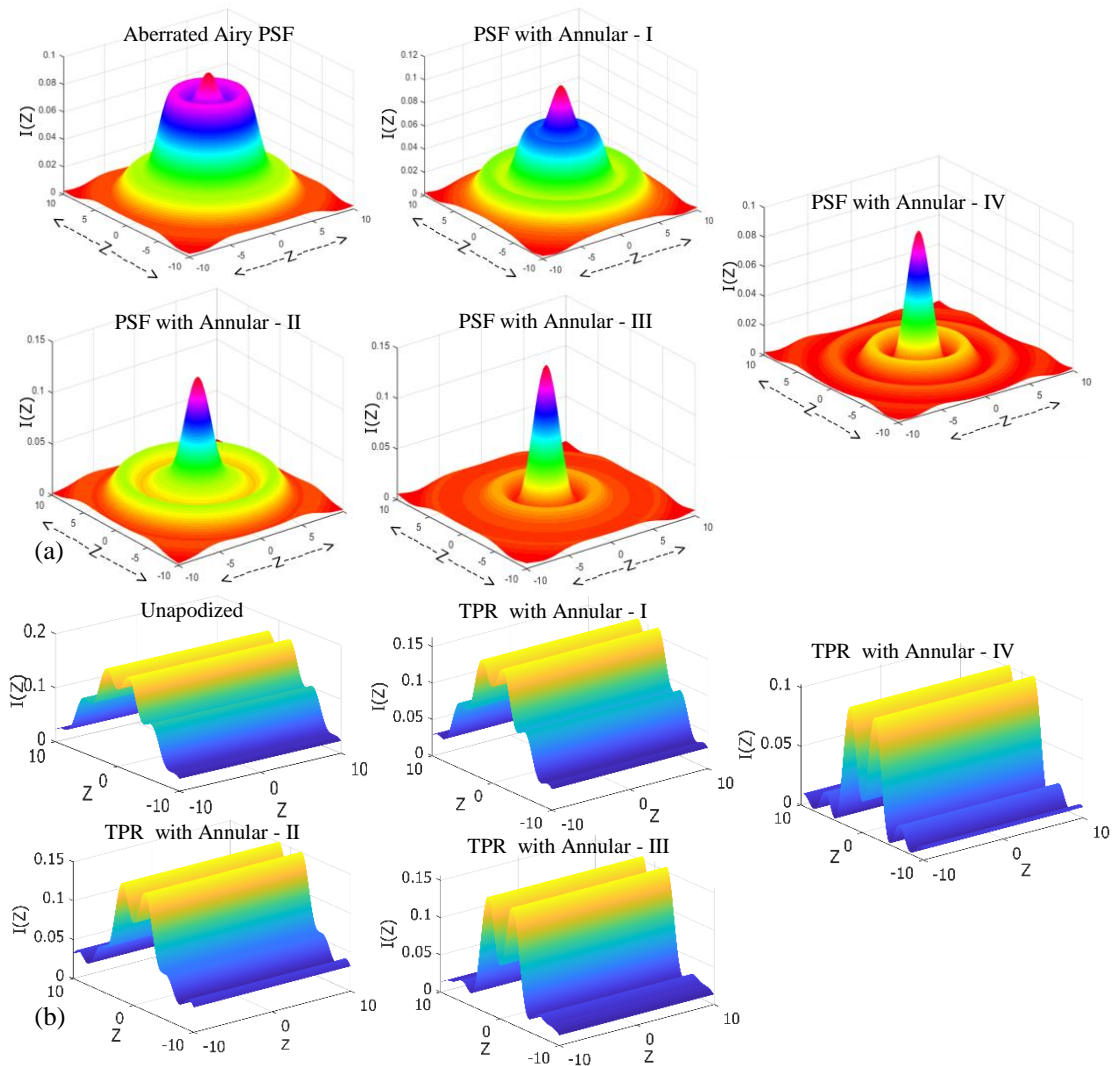


Fig 2. (a) PSF intensity distributions formed by different truncated apodizers in the presence of high defocusing effect ($\phi_d = 2\pi$) and spherical aberration ($\phi_s = 2\pi$) (b) Corresponding composite intensity distributions from two mutually incoherent point sources separated by the distance is equal to the Rayleigh limit ($1.22\lambda / D$), D - Aperture diameter, TPR - Two-point resolution. The truncated apodizers investigated in the form of Annular-I [$\beta=0.5$, $\varepsilon=0.2$], Annular-II [$\beta=0.5$, $\varepsilon=0.4$], Annular-III [$\beta=0.5$, $\varepsilon=0.6$], and Annular-IV [$\beta=0.5$, $\varepsilon=0.8$].

The resultant amplitude functions of the two incoherent point sources are measured from equation (4). ‘ μ ’ controls the degree of the spatial coherence of illumination, and μ is equal to zero. In the present study, the two-point sources are separated by the distance $2W=Z_0$ is equal to the incoherent Rayleigh limit ($3.832=1.22\lambda / D$) [1].

III. RESULTS AND DISCUSSION

Fig. 1 shows that the selection of truncated apertures for modifying the axial and lateral distribution of the point spread function (PSF) depends on the degree of defocusing effect introduced in the optical system. Fig. 1(a) shows the Airy PSF formed by the aberration-free optical system is compared with the PSF obtained when the clear aperture is truncated ($\epsilon=0.2$, 20%) by the amplitude mask. In this case, there are no significant changes observed in the resultant PSF distribution obtained in the Gaussian focal plane, as seen in Fig. 1(b). Fig. 1(c) and Fig. 1(d) illustrate PSF intensity distributions formed by the defocused optical system, and the degree of defocusing effect is presented as $\phi_d = 2\pi$. With the defocusing, the light flux enclosed in the central lobe is displaced into the sidelobes region. The resultant PSF distribution is found with non-zero first minima, and the intensity levels of sidelobes are relatively high (see Fig. 1(c)). Here, to reshape the deformed PSF formed by the defocused optical system, the aperture with $\epsilon = 0.4$ is considered. The PSF profile is reshaped in the defocused plane ($\phi_d = 2\pi$), where the sidelobes are flattened, and the first-minima is found with zero intensity, as shown in Fig. 1(d).

Figs. 1(e)-1(h) depict the PSF intensity distributions formed by the optical system with an extreme defocusing effect. For the defocused plane $\phi_d = 3\pi$, the PSF intensity profiles are highly deformed (Fig. 1(e)). The sidelobe intensities are relatively high as a significant part of the central lobe energy is longitudinally displaced, and the PSF with non-zero first minima (Fig. 1(e)). With the truncated apodizer $\epsilon = 0.6$ (60%), the PSF in the extreme defocused plane has found with suppressed sidelobes and zero first-minima. It has shown that the axial and lateral resolution of the PSF is engineered and improved (Fig. 1(f)). For the defocused plane $\phi_d = 4\pi$, the resolution of the PSF has highly deteriorated. It is observed that the intensity of sidelobes is higher than the central lobe intensity, and the non-zero first minima position is vertically shifted more (Fig. 1(g)). By employing the truncated aperture with $\epsilon=0.8$ (80%), the PSF intensity distribution formed by the defocused optical system is dramatically improved and modified, and it is evidenced in Fig. 1(h). Next, the performance of the optical system in the presence of high defocusing and the primary spherical aberrations for various truncated apodizers is investigated, and the results are shown in Fig. 2.

Fig. 2(a) represents highly aberrated Airy PSF, in which the internal energy of the central lobe is displaced results in enhanced sidelobes and distorted PSF. For $\beta = 0.5$, with the increase in the amount of truncation in the apodizer ($\epsilon = 0.0$ to 0.8), the central lobe of the PSF becomes sharpened, and sidelobes are suppressed to zero-intensity levels. The resulting 3D PSF profile curves have shown in Fig. 2(a), titled PSF with Annular-I to PSF with Annular-IV. In the presence of the same truncated apodizers, the resolution of two overlapping point sources is also investigated, and the corresponding intensity distributions are shown in Fig. 2(b).

In Fig. 2(b), the value $Z_0 = 3.832$ (incoherent Rayleigh limit), which is a distance separation. The two overlapping

point sources that mutually incoherent are well-resolved for the values of ϵ varying from 0 to 0.8, as is evidenced by the dips in the composite intensity distributions. As the truncation at the center of the apodizer increases, the two central lobes in the resulting composite intensity distribution move outwards, forming a clear measured separation much larger than the separation of two-point images found in unapodized case, as shown in Fig. 2(b). The two-point resolution is excellent with the truncated apodizer $\beta=0.5$ and $\epsilon=0.8$, the resulting intensity curve shown in Fig. 2(b) with the title Annular-IV. The presence of a steep dip with low-intensity evidence that the two-point resolution of the aberrated optical system increased dramatically with the truncation effect in the apodizer. Note that the image intensity decreases as the truncation (ϵ) effect increases (Fig. 2(b)).

IV. CONCLUSION

In the present study, the truncated apodizers with higher ϵ values have shown remarkable performance in engineering the PSF formed by the optical system underneath a high defocusing effect and the primary spherical aberrations. The incoherent two-point resolution rapidly increases with the apodizer when the truncation $\epsilon > 0.4$. Furthermore, the truncated apodizers can increase the two-point resolution even under the partially, fully coherent illumination.

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